

High order methods in Numerical Relativity

Florian Atteneder

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1. Project: Entropy viscosity method

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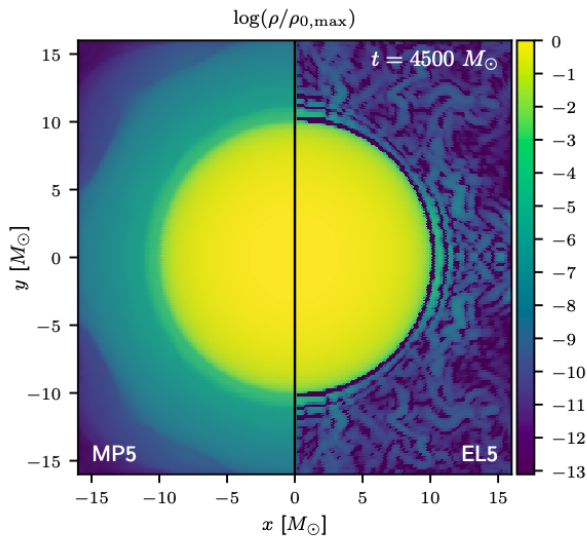


Figure 1: Taken from [1]

How does it work? [2], [3]

Conservation law

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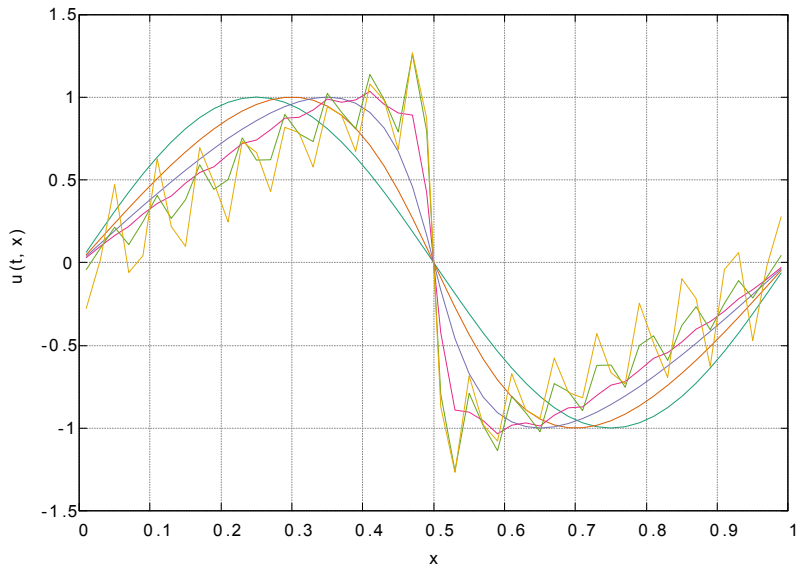
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Tuning via

$$\nu = \nu(c_{max}, c_e), \quad c_{max}, c_e \in \mathbb{R}$$

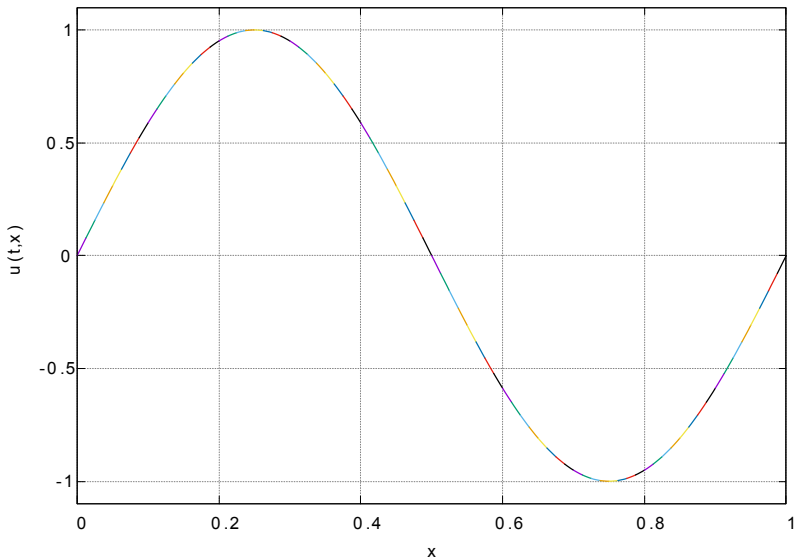
Burgers' equation (gone bad)

Viscous Burgers' equation $u_t + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$, Npts = 50



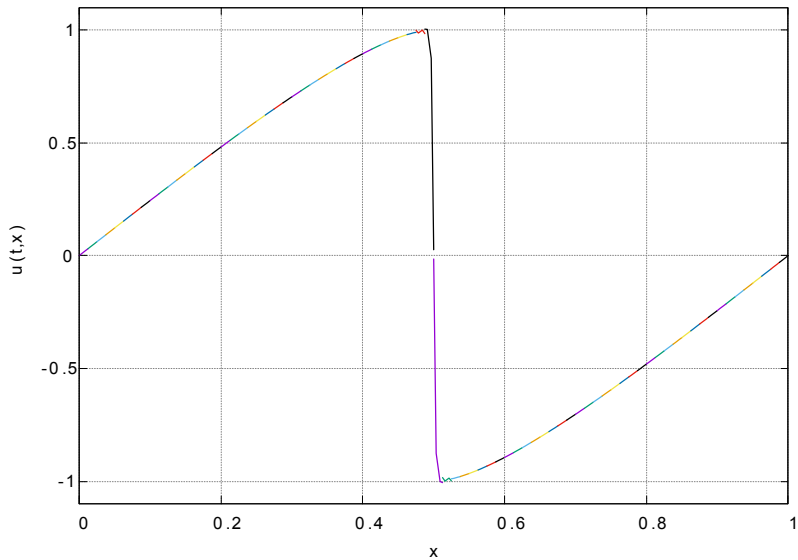
Burgers' equation $f(u) = u^2$

LGL grid, $N = 3$, $K = 80$, $T = 0.0$, $ce = 0.5$, $cmax = 1.0$



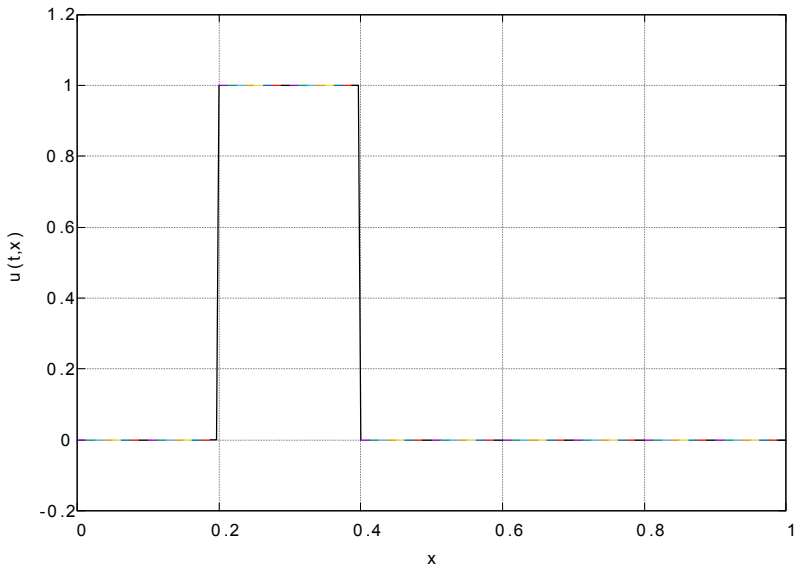
Burgers' equations $f(u) = u^2$

LGL grid, $N = 3$, $K = 80$, $T = 0.25$, $ce = 0.5$, $cmax = 1.0$



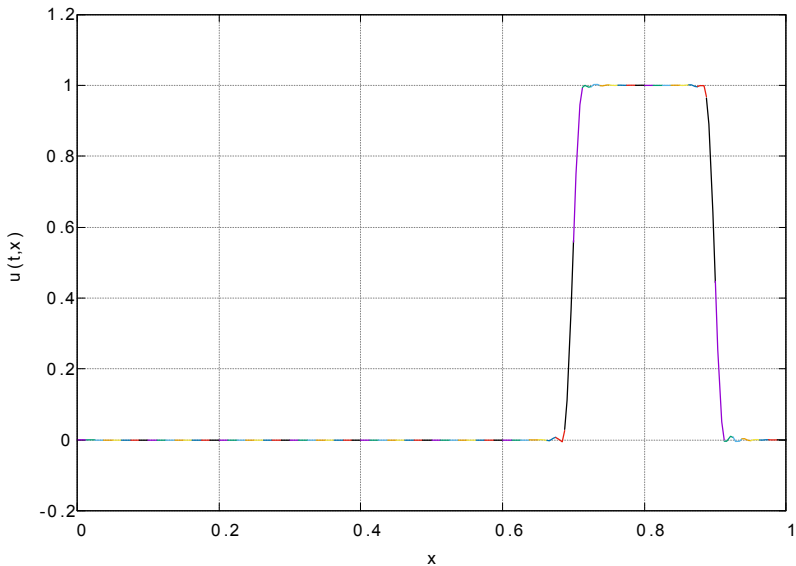
Advection equation $f(u) = u$

LGL grid, $N = 3$, $K = 80$, $T = 0.0$, $ce = 0.05$, $cmax = 0.01$



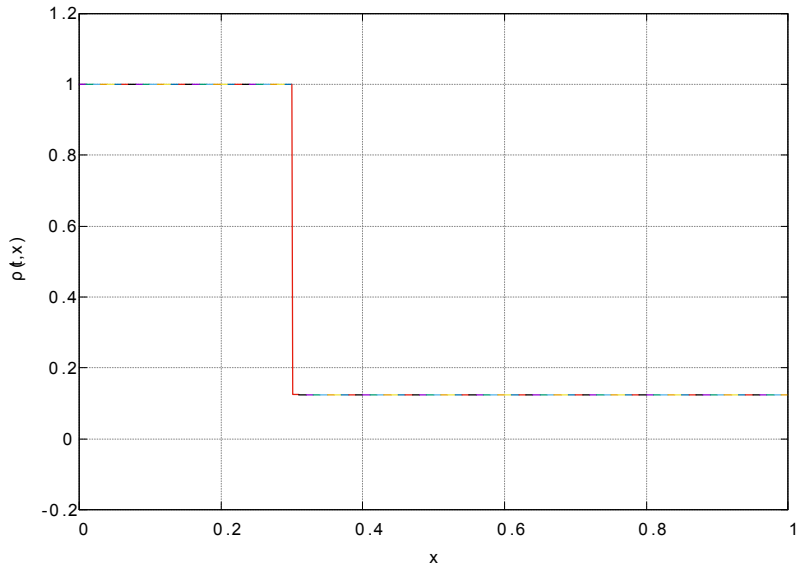
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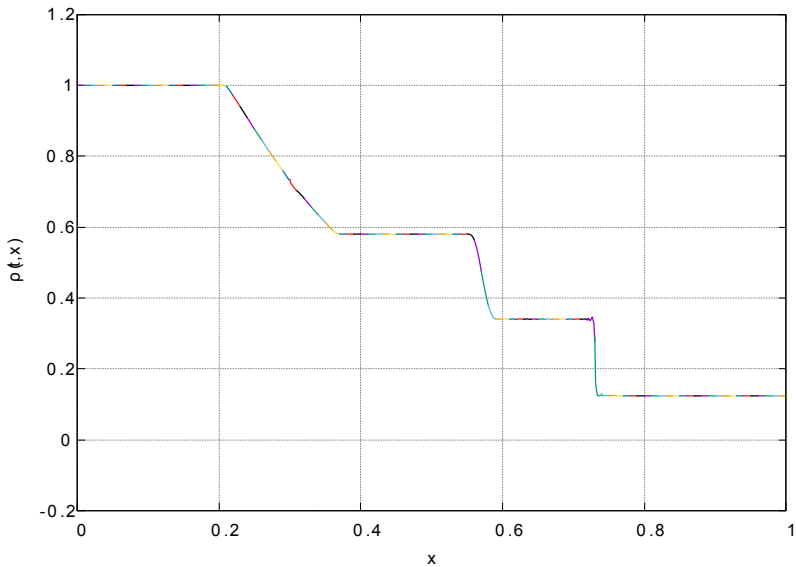
Euler equations

LGL grid, $N = 5$, $K = 100$, $T = 0.0$, $ce = 1.0$, $cmax = 0.1$



Euler equations

LGL grid, $N = 5$, $K = 100$, $T = 0.2$, $ce = 1.0$, $cm_{ax} = 0.1$



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Thank you!

Bibliography

- [1] F. Guercilena, D. Radice, and L. Rezzolla, “Entropy-limited hydrodynamics: A novel approach to relativistic hydrodynamics,” *Computational Astrophysics and Cosmology*, vol. 4, no. 1, p. 3, 2017.
- [2] J.-L. Guermond and R. Pasquetti, *Entropy-based nonlinear viscosity for fourier approximations of conservation laws*, vol. 346. Elsevier, 2008, pp. 801–806.
- [3] V. Zingan, J.-L. Guermond, J. Morel, and B. Popov, “Implementation of the entropy viscosity method with the discontinuous galerkin method,” *Computer Methods in Applied Mechanics and Engineering*, vol. 253, pp. 479–490, 2013.